

Cosmology with x-matter

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ABSTRACT

Motivated by the possibility of $H_0 t_0 > 1$ where H_0 and t_0 are the Hubble parameter and the age of the universe, respectively, we investigate the cosmology including x-matter. x-matter is expressed by the equation of state $p_x = w_0 \rho_{x0} + c_s^2(\rho_x - \rho_{x0})$, where p_x , ρ_x and ρ_{x0} are the pressure, the density of x-matter and the density at present, respectively. w_0 and c_s^2 are functions of ρ_x in general. x-matter has the most general form of the equation of state which is characterized by 1) violation of strong energy condition at present for $w_0 < -1/3$; 2) locally stable (i.e. $c_s^2 \geq 0$); 3) causality is guaranteed ($c_s \leq 1$). Considering the case that w_0 and c_s^2 are constants, we find that there is a large parameter space of $(w_0, c_s^2, \Omega_{x0})$ in which the model universe is consistent with the age of the universe and the observations of distant Type I supernovae.

Key words: cosmology: theory – dark matter.

1 INTRODUCTION

The age of the globular clusters 15 ± 2 Gyr (Chaboyer et al. 1995) and the age of the universe with current $H_0 = 70 \pm 10$ km/s/Mpc (Freedman 1996; Riess et al. 1995, 1996) may suggest $H_0 t_0 > 1$ where H_0 and t_0 are the Hubble parameter and the age of the universe, respectively.* $H_0 t_0 > 1$ stands for the apparent contradiction of the age of the universe; the universe is younger than the oldest globular cluster. For this age problem Nakamura et al. (1995) proved the theorem such that if $H_0 t_0 > 1$, either 1) Einstein’s theory of gravity is not correct theory of gravitation, 2) the strong energy condition is not satisfied, or 3) foliation by geodesic slicing does not exist.

For the age problem of the universe, usually the existence of the cosmological term is suggested. However, recent SNIa survey by Perlmutter et al. (1995, 1997) and the statistics of gravitational lensing (Kochanek 1996) are giving some doubts on the Λ -dominated models. From the point view of the above general theorem (Nakamura et al. 1995) the constant cosmological term is nothing but an example to save the age problem. For instance, the model with the scalar field which works as a time varying cosmological term was proposed based on particle physics (Fujii 1989) and investigated the role in cosmology (Peebles & Ratra 1988; Sato, Terasawa & Yokoyama 1989; Sugiyama & Sato 1992). Re-

cently, Steinhardt (1996) suggested an alternative to lambda term in which a matter with $p = w\rho$ ($-1 < w < 0$) resides in addition to ordinary CDM.

Turner and White (1997) have done comprehensive study of this so-called xCDM model. In their paper, they mentioned that they do not allow for clumping of xCDM because a component with $w < 0$ is highly unstable to growth of perturbations on small scales. Therefore they assumed xCDM to be a smooth component on small scales. They suggested kinds of topological defects may behave like such “xCDM”. However, $w \equiv p/\rho < 0$ does not necessarily imply $c_s^2 \equiv \delta p/\delta \rho < 0$ which causes the instability. Thus we may construct a clumpy xCDM model such that $w < 0$ and $c_s^2 \geq 0$ if we can find proper energy momentum tensor and equation of state. As shown on pp.89 of Hawking and Ellis (1973), the energy momentum tensor is classified into four types. The form of the energy momentum tensor compatible with the global isotropy of the universe should be of Type I with pressure $p_1 = p_2 = p_3 = p$. In this letter, we shall study the general cosmology, in which x-matter with $w < 0$ and $c_s^2 \geq 0$ resides in addition to ordinary cold dark matter (CDM), baryonic matter and radiation. This kind of general study may give us some insights into the age problem.

2 BASICS

As mentioned in §1, x-matter should possess following nature, i.e., $w \equiv p_x/\rho_x < -1/3$ to violate strong energy condition and $c_s^2 \equiv \delta p/\delta \rho \geq 0$ to stabilize growth of perturbations. The former condition is necessary (though not sufficient) to

* The recent Hipparcos results (Feast & Catchpole 1997; Reid 1997) suggest the globular cluster age 12 Gyr and $H_0 \simeq 65$ km/s/Mpc, and then $H_0 t_0 \gtrsim 0.82$. The age problem, however, still persists if $\Omega_{M0} \gtrsim 0.27$.

elongate the cosmic age (Nakamura et al. 1995). Several matter fields are known to violate the strong energy condition: a massive scalar field, a domain wall and the cosmological constant. c_s represents, as shown, the sound velocity of x-matter and determines how the density evolves. Together with the latter condition, we demand that it is not faster than the light velocity. As the simplest form, we employ the equation of state of x-matter as

$$p_x = w_0 \rho_{x0} + c_s^2(\rho_x - \rho_{x0}), \quad (1)$$

where the suffix 0 denotes the present value. Although w_0 and c_s are functions of ρ_x in general, to simplify the discussion we assume that they are constants in this letter. Our x-matter model includes the cosmological constant model as a special case since $p_x = -\rho_{x0}$ and $\rho_x = \rho_{x0}$ for $w_0 = -1$. On the other hand, xCDM by Turner and White(1997) is not included since in xCDM $c_s^2 < 0$ so that it is unstable for all scale perturbations. According to above requirements, we consider the region $-1 \leq w_0 < -1/3$, $0 \leq c_s^2 \leq 1$, and $\rho_x \geq 0$. A sketch of the equation of state is shown in Fig.1.

The energy equation is

$$\frac{d\rho_x}{da} = -\frac{3}{a}(\rho_x + p_x), \quad (2)$$

where a is the scale factor. Normalizing a as $a_0 = 1$, we have

$$\rho_x = \frac{\rho_{x0}}{1 + c_s^2} \left((1 + w_0)a^{-3(1+c_s^2)} + c_s^2 - w_0 \right). \quad (3)$$

We note that the cosmological constant model is an asymptotic limit in our x-matter model: $p_x/\rho_x \rightarrow -1$ as $a \rightarrow \infty$.

The Friedmann equation is

$$\begin{aligned} \frac{1}{H_0^2} \left(\frac{\dot{a}}{a} \right)^2 + \frac{\Omega_{M0} + \Omega_{x0} - 1}{a^2} \\ = \frac{\Omega_{M0}}{a^3} + \frac{\Omega_{r0}}{a^4} \\ + \frac{\Omega_{x0}}{1 + c_s^2} \left((1 + w_0)a^{-3(1+c_s^2)} + c_s^2 - w_0 \right), \end{aligned} \quad (4)$$

where Ω_{M0} , Ω_{r0} and Ω_{x0} are the present density parameter of CDM, radiation (photon and neutrino) and x-matter, respectively and over dot denotes time derivative.

Let us consider first the constraint on c_s^2 . This comes from the requirement that the process of Big Bang Nucleosynthesis (BBN) is not appreciably disturbed by the existence of x-matter. Eq.(3) shows that x-matter grows faster than dust matter as a becomes smaller. If $c_s^2 > 1/3$, it does even faster than radiation and would dominate over radiation. In order that the density of x-matter is smaller than the one of a single massless neutrino species at BBN ($T_{\text{BBN}} \simeq 0.1\text{MeV}$),

$$\Omega_{x0} \frac{1 + w_0}{1 + c_s^2} < 1.1 \times 10^{-5} \left(\frac{0.7}{h} \right)^2 a_{\text{BBN}}^{-1+3c_s^2}, \quad (5)$$

where $a_{\text{BBN}} = 2.3 \times 10^{-9} (T_{\text{BBN}}/0.1\text{MeV})$ is the scale factor at BBN and h is the non-dimensional Hubble parameter normalized by 100km/s/Mpc. Constraints on the $w_0 - c_s^2$ plane are shown in Fig.2. From this figure, we can conclude that $c_s^2 \lesssim 0.15$ should be required.

The typical evolution of densities are shown in Fig.3 for $c_s^2 = 0$ and 0.2. In the early universe x-matter is essentially the same as CDM for $c_s^2 = 0$ but exceeds CDM for $c_s^2 > 0$. On the other hand, for $a > 0.1$ where the dominant contribution

to the age of the universe is made, it is different from CDM regardless of the value of c_s^2 as is shown in Fig.3.

3 COSMOLOGY

Now let us develop the cosmology with x-matter. Hereafter we consider the universe during the matter-dominated era.

3.1 Critical Values

Likewise the cosmology with the cosmological constant, there are some critical values of Ω_{x0} which correspond to the boundary between the universe with or without the big-bang; the boundary between the recollapsing universe and the expanding universe.

The Friedmann equation (4) can be read as the “energy conservation equation”, the kinetic energy term being \dot{a}^2/H_0^2 , the potential energy term $V(a)$ being

$$V(a) = -\Omega_{M0}a^{-1} - \frac{\Omega_{x0}}{1 + c_s^2} \left((1 + w_0)a^{-1-3c_s^2} + (c_s^2 - w_0)a^2 \right), \quad (6)$$

and the total energy E being $E = 1 - \Omega_{M0} - \Omega_{x0}$. The universe can recollapse or bounce if $E \leq \max(V(a))$. For $c_s^2 = 0$, the maximum of $V(a)$ is evaluated analytically

$$V(a) \leq -\frac{3}{2} (\Omega_{M0} + (1 + w_0)\Omega_{x0})^{2/3} (-2w_0\Omega_{x0})^{1/3}. \quad (7)$$

Therefore the condition $E \leq \max(V(a))$ reads

$$4(1 - \Omega_{M0} - \Omega_{x0})^3 \leq 27w_0\Omega_{x0} (\Omega_{M0} + (1 + w_0)\Omega_{x0})^2. \quad (8)$$

The equality, which is a cubic equation of Ω_{x0} , decides the critical values of Ω_{x0} .

Fig.4 shows the critical $\Omega_{xc1}, \Omega_{xc2}$ for $c_s^2 = 0$ and several w_0 . If $\Omega_{x0} > \Omega_{xc1}$, then the universe bounces without the big-bang; if $\Omega_{xc2} < \Omega_{x0} < \Omega_{xc1}$, the universe with big-bang expands forever; if $\Omega_{x0} < \Omega_{xc2}$, recollapses. The difference between bounce and recollapse comes from the value of a ($=a_m$) at the maximum value of $V(a)$ (i.e. whether $a_m < 1$ or $a_m > 1$).

3.2 Age of the Universe

The cosmic age is given by

$$\begin{aligned} H_0 t_0 &= \int_0^1 \frac{da}{a} \left[\Omega_{M0} a^{-3} + (1 - \Omega_{M0} - \Omega_{x0}) a^{-2} \right. \\ &\quad \left. + \frac{\Omega_{x0}}{1 + c_s^2} \left((1 + w_0) a^{-3(1+c_s^2)} + c_s^2 - w_0 \right) \right]^{-1/2}. \end{aligned} \quad (9)$$

The functional form of $H_0 t_0$ shows that it is a decreasing function of c_s^2 or w_0 . The current constraints on $h = 0.70 \pm 0.10$ (Freedman 1996; Riess et al. 1995, 1996) and the globular cluster age $15 \pm 2\text{Gyr}$ (Chaboyer et al. 1995) suggest at least $H_0 t_0 \geq 0.80$. In Figs.5, the contour plots of $H_0 t_0$ in the $(\Omega_{M0}, \Omega_{x0})$ plane are shown. For fixed Ω_{M0} , $H_0 t_0$ is an increasing function of Ω_{x0} in Figs.5(b), 5(d), 5(f) and 5(g), while it is a decreasing one in Figs. 5(a) and 5(c). In Fig.5(e) it is either an increasing or a decreasing function, depending on Ω_{M0} . The reason for this behavior can be understood well by differentiating equation (9) by Ω_{x0} as

$$\frac{\partial H_0 t_0}{\partial \Omega_{x0}}$$

$$\begin{aligned}
&= \int_0^1 \frac{da}{2a} \left(a^{-2} - \frac{1}{1+c_s^2} ((1+w_0)a^{-3(1+c_s^2)} + c_s^2 - w_0) \right) \\
&\times \left[\Omega_{M0} a^{-3} + (1 - \Omega_{M0} - \Omega_{x0}) a^{-2} \right. \\
&\left. + \frac{\Omega_{x0}}{1+c_s^2} \left((1+w_0)a^{-3(1+c_s^2)} + c_s^2 - w_0 \right) \right]^{-3/2}. \quad (10)
\end{aligned}$$

For $w_0 = -1$, the sign of $\partial H_0 t_0 / \partial \Omega_{x0}$ is determined by the sign of $a^{-2} - 1 > 0$ so that $H_0 t_0$ is an increasing function of Ω_{x0} . This also explains the behaviors for $w_0 = -0.8$ as follows. With the increase of c_s , the relative importance of the negative term in the expression of $\partial H_0 t_0 / \partial \Omega_{x0}$ increases so that $H_0 t_0$ becomes a slowly increasing function of Ω_{x0} . As c_s is increased, x-matter behaves like radiation so that $H_0 t_0$ should approach the value of the radiation case. Note here $H_0 t_0 = 0.5$ in the flat radiation dominant universe. For $w_0 = -0.6$, on the other hand, $H_0 t_0$ is almost independent of Ω_{x0} . This shows that x-matter is ineffective to increase $H_0 t_0$ considerably for $w_0 > -0.6$. It is easy to show that for $w_0 > -1/3$ x-matter does not save the age problem at all, which is already proved by the general theorem (Nakamura et al. 1995).

3.3 Luminosity Distance

We consider the luminosity distance-redshift relation in x-matter models. The luminosity distance is defined by

$$\begin{aligned}
H_0 d_L(z) &= (1+z) \int_0^z dz' \left[\Omega_{M0} (1+z')^3 \right. \\
&+ (1 - \Omega_{M0} - \Omega_{x0}) (1+z')^2 \\
&\left. + \frac{\Omega_{x0}}{1+c_s^2} \left((1+w_0)(1+z')^{3(1+c_s^2)} + c_s^2 - w_0 \right) \right]^{-1/2}. \quad (11)
\end{aligned}$$

Incidentally, the deceleration parameter q_0 is given by $q_0 = \Omega_{M0}/2 + (1+3w_0)\Omega_{x0}/2$. The luminosity distance-redshift relation differs for two cosmological models with the same q_0 and H_0 ; it depends on the equation of state of x-matter. The current uncertainty at 95%(68%) C.L. in the distance to SNIa at $z \simeq 0.4$ is $0.38(0.40) < H_0 d_L(z=0.4) < 0.51(0.48)$ (Perlmutter et al. 1995, 1997; Turner & White 1997).

In Fig.6 regions of w_0 and Ω_{x0} which satisfy both $H_0 t_0 > 0.80$ and $0.38 < H_0 d_L(z=0.4) < 0.51$ are shown. A region bounded by a solid line is for $\Omega_{M0} = 0.1$, a dotted line for $\Omega_{M0} = 0.3$, a dashed line for $\Omega_{M0} = 0.5$. Upper lines come from the age limit which can be easily understood from the argument in the previous subsection. Lower right lines mean that too much Ω_{x0} increases the luminosity distance above the observational limit $H_0 d_L(z=0.4) < 0.51$. We find that large parameter spaces are left even for a closed universe model as well as an open universe model. The Λ -dominated universe corresponds to the $w_0 = -1$ line in Fig.6. It is interesting to note that high $\Omega_{M0} (> 0.3)$ cosmological models are allowed in the x-matter model although excluded in the Λ -dominated universe.

4 SUMMARY

We have investigated the possibility that the dark matter component has the equation of state $p_x = w_0 \rho_{x0} + c_s^2(\rho_x - \rho_{x0})$ such that $-1 \leq w_0 < -1/3$ and $0 \leq c_s^2 \leq 1$. We have

studied various limits on w_0 , c_s^2 and Ω_{x0} . We have found that there is a large parameter space of $(w_0, c_s^2, \Omega_{x0})$ in which the model universe is consistent with the age of the universe and the observations of distant Type I supernovae. It may not be so meaningful to assess the “best fit” values of w_0, c_s^2, Ω_{x0} because observational data will be updated soon and the situation is not settled yet. It is interesting to study the properties of the large scale structure in x-matter models. We intend to extend a detailed study of perturbation theory and “neoclassical” tests of x-matter models. Those include the statistics of gravitational lensing and spectra of the cosmic microwave background radiation. The results will be reported in a future publication (Chiba, Sugiyama, and Nakamura, work in progress).

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FIGURE CAPTIONS

Figure 1: A sketch of the equation of state of “x-matter”. The solid line shows $p_x = w_0 \rho_{x0} + c_s^2(\rho_x - \rho_{x0})$ and the dotted line shows $p_x = w_0 \rho_{x0}$.

Figure 2: Constraints from BBN on the $w_0 - c_s^2$ plane. The lines show that the energy density of x-matter is equal to the one of a single species neutrino for $\Omega_{x0} = 0.5, 0.7$ and 0.9 .

Regions above these lines are excluded. $T_{BBN} = 0.1\text{MeV}$ and $h = 0.7$ are employed.

Figure 3: The evolution of densities ρ_x, ρ_M (matter density), ρ_γ (radiation density) for $c_s = 0$ and $0.2, w_0 = -0.8, \Omega_{M0} = 0.1$, and $\Omega_{x0} = 0.9$. The scale factor is normalized as $a = 1$ at the present.

Figure 4: The critical values of Ω_{x0} for $w_0 = -1, -0.8, -0.6$ are shown. Upper lines correspond to Ω_{xc1} and lower lines correspond to Ω_{xc2} (see text for their definition). For $\Omega_{x0} > \Omega_{xc1}$, the universe bounces without the big-bang; for $\Omega_{xc2} < \Omega_{x0} < \Omega_{xc1}$, the universe with big-bang expands forever; $\Omega_{x0} < \Omega_{xc2}$, recollapses. The difference between bounce and recollapse comes from the value of a ($=a_m$) at the maximum value of $V(a)$ (i.e. whether $a_m < 1$ or $a_m > 1$).

Figure 5: The contour of $H_0 t_0$ for $c_s^2 = 0, 0.1, 0.15$ and $w_0 = -1, -0.8, -0.6$. A dotted line corresponds to a flat universe. Note that the $w_0 = -1$ case is independent of c_s .

Figure 6: Allowed regions of Ω_{x0} and w_0 for $\Omega_{M0} = 0.1, 0.3, 0.5$ and $c_s^2 = 0, 0.1, 0.15$ when $H_0 t_0 > 0.8$, $H_0 t_0 > 0.9$, or $H_0 t_0 > 1.0$. A region bounded by a solid line is for $\Omega_{M0} = 0.1$, a dotted line for $\Omega_{M0} = 0.3$, a dashed line for $\Omega_{M0} = 0.5$.

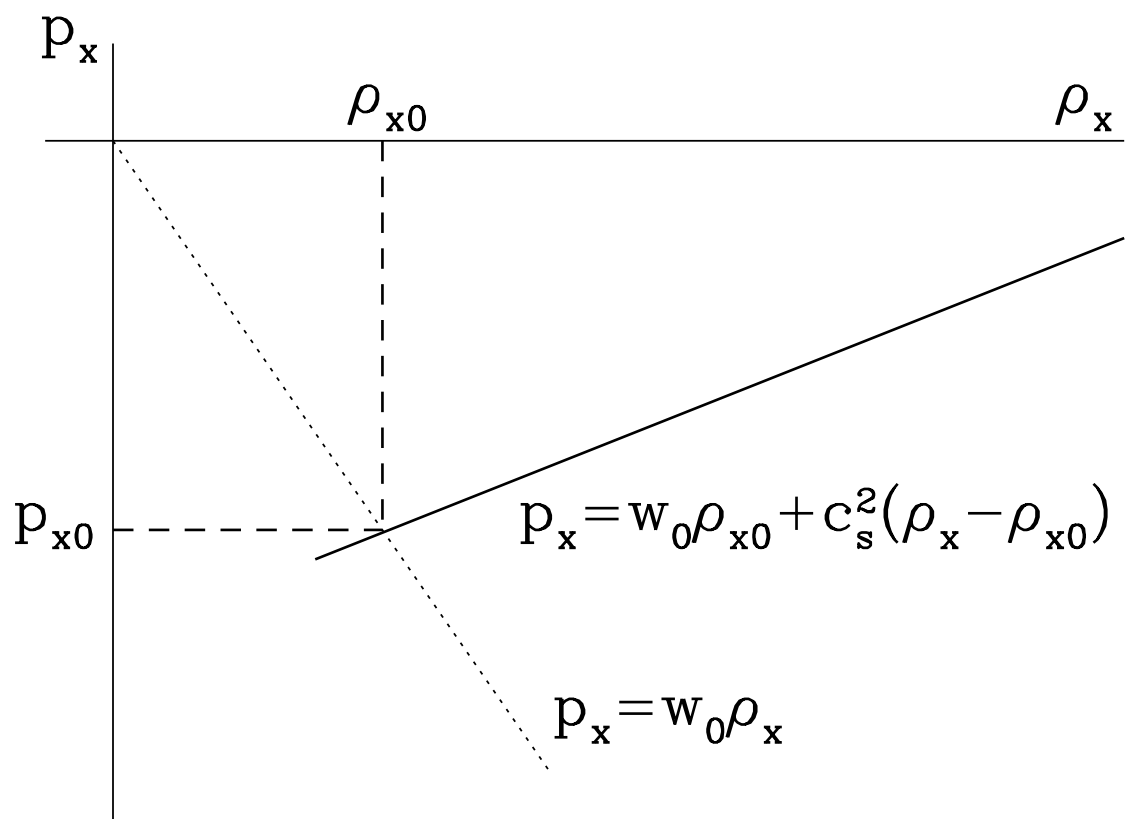


Fig. 1

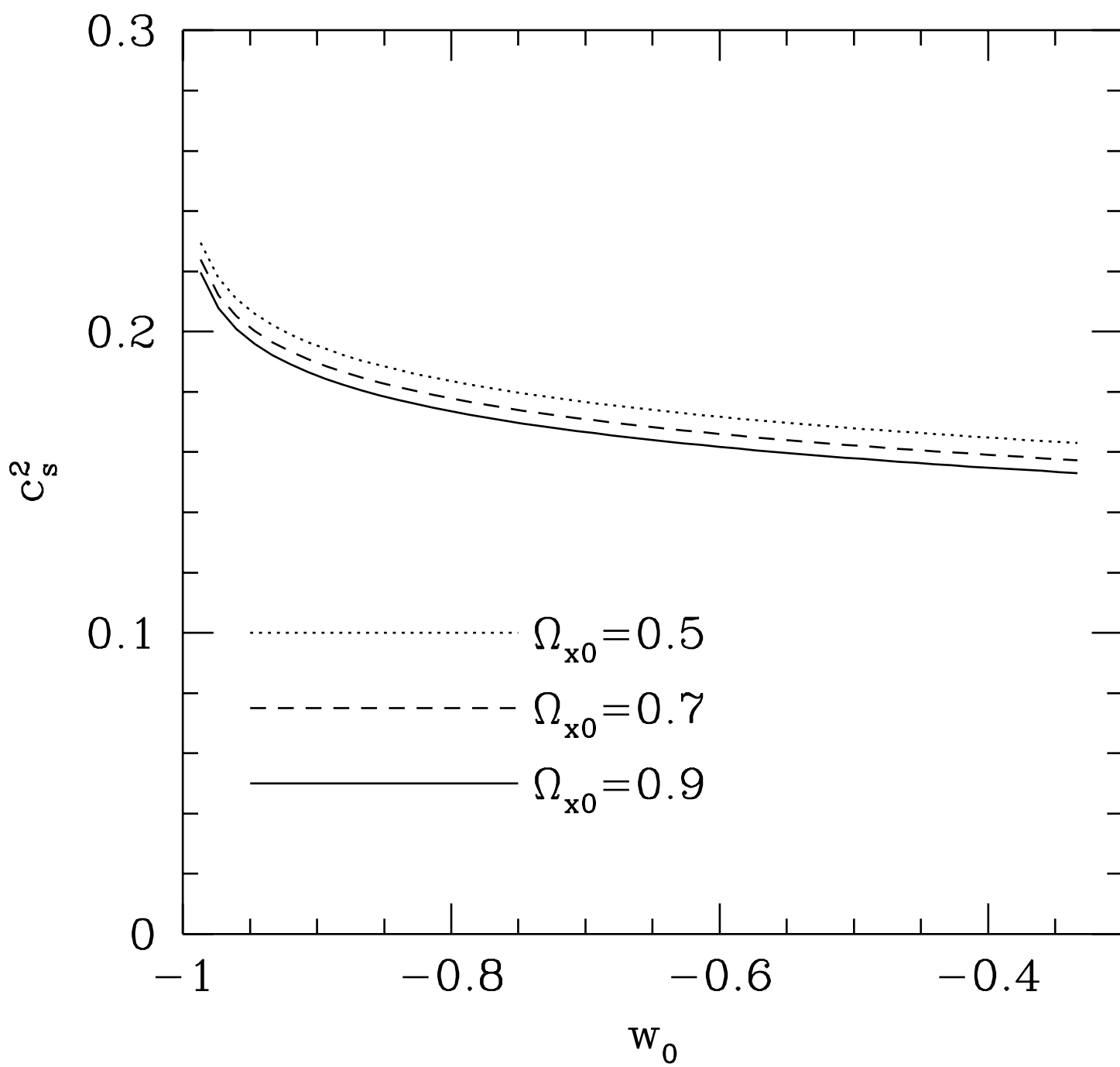
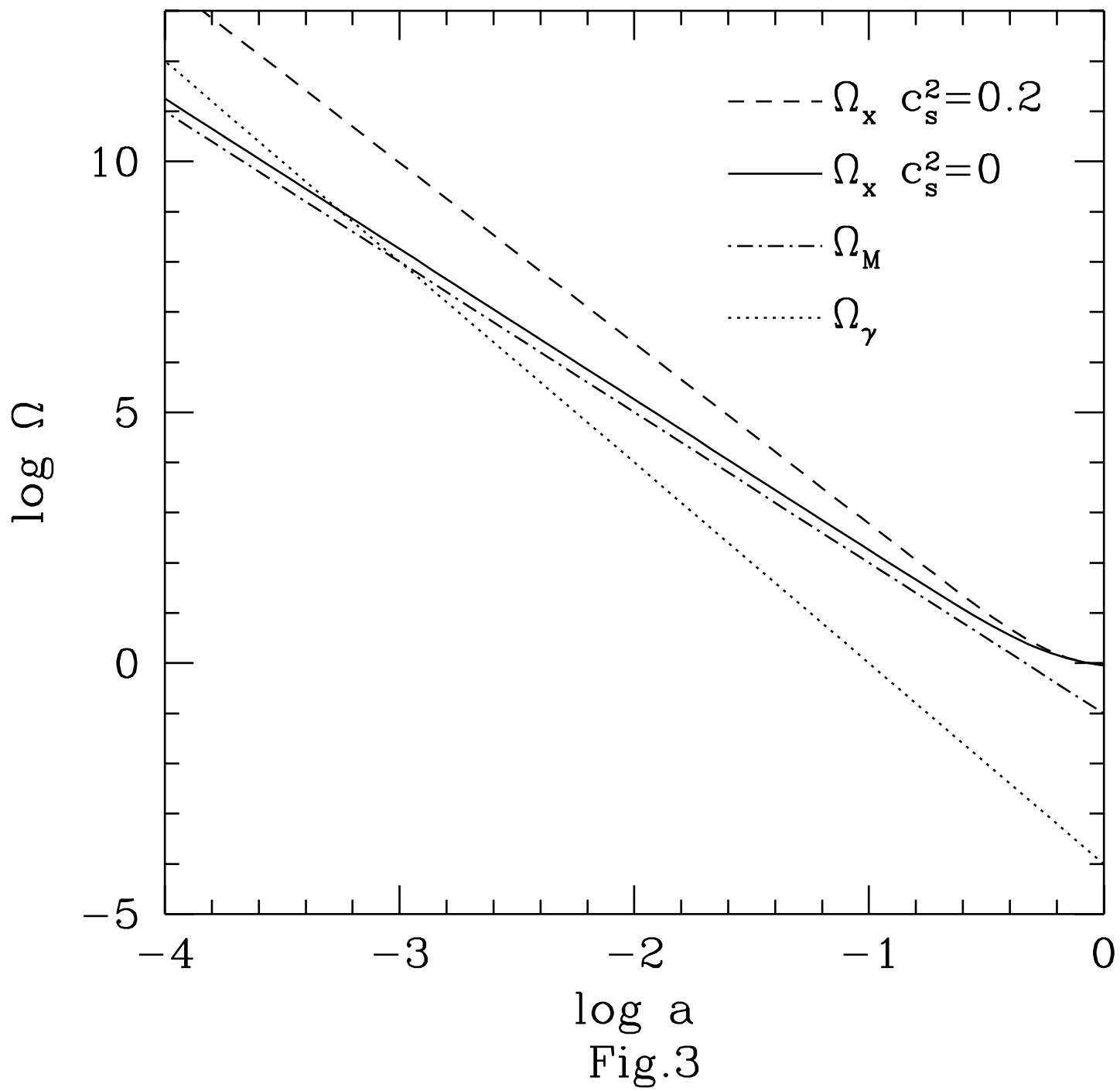
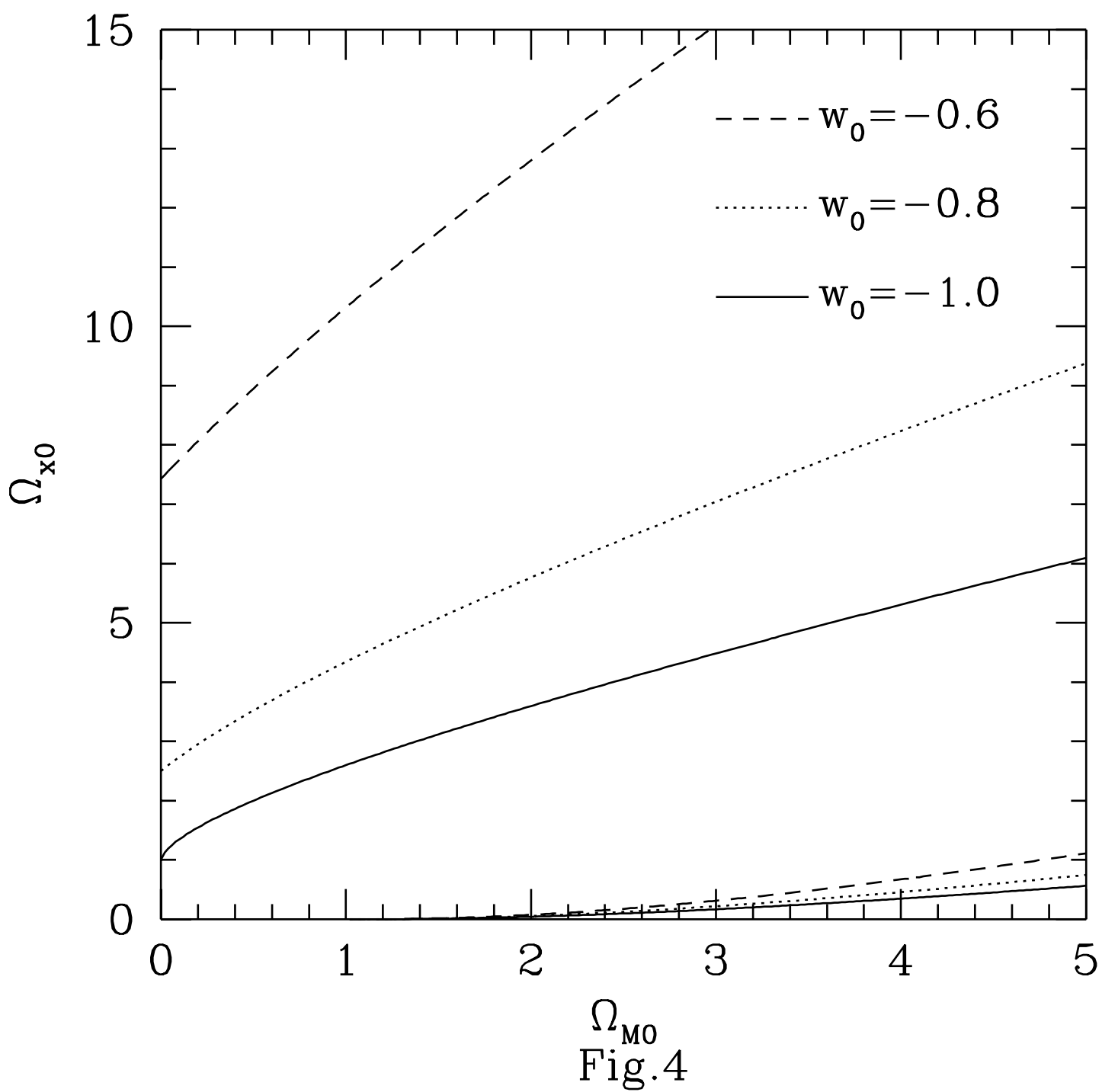


Fig.2





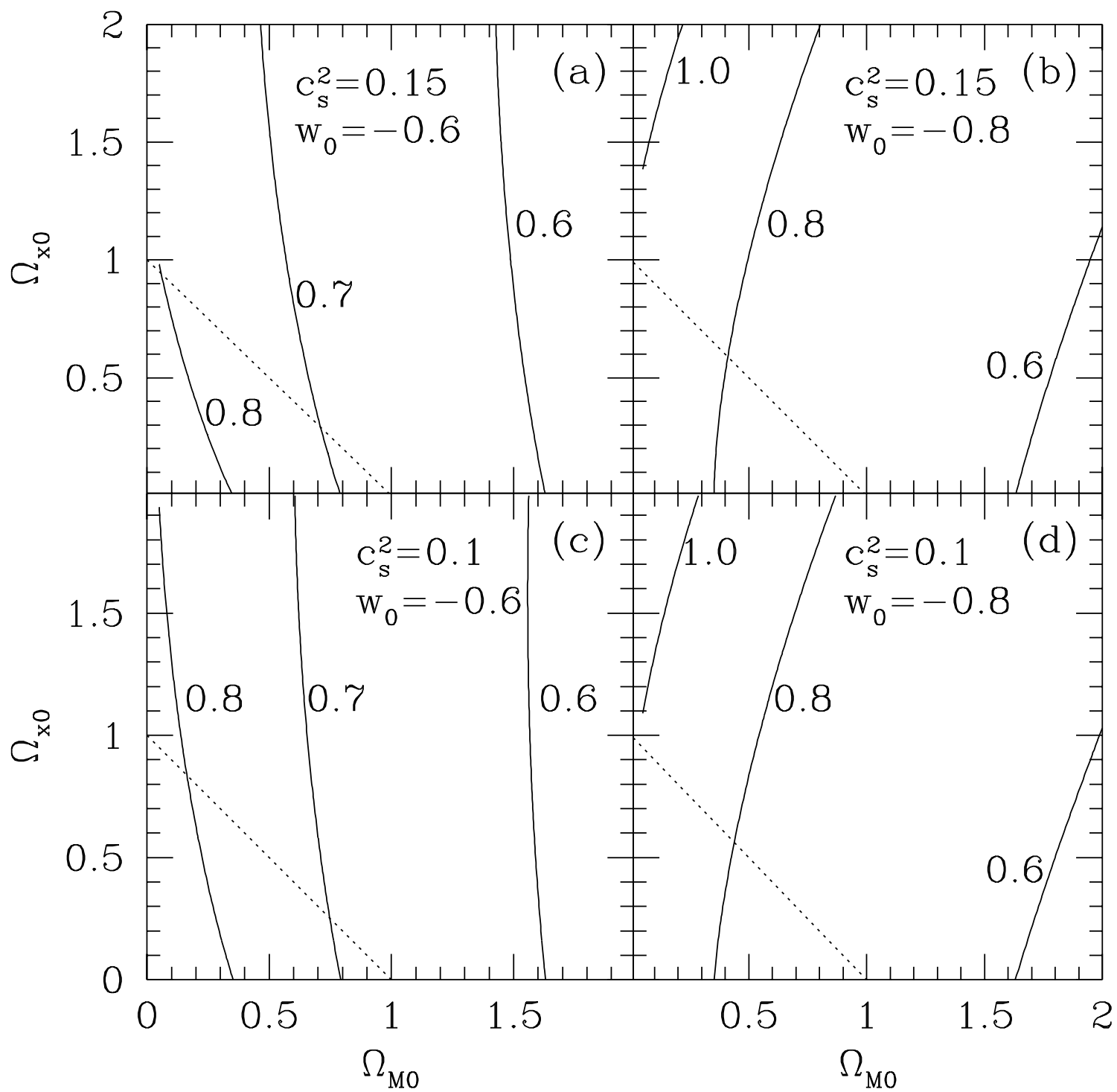


Fig. 5

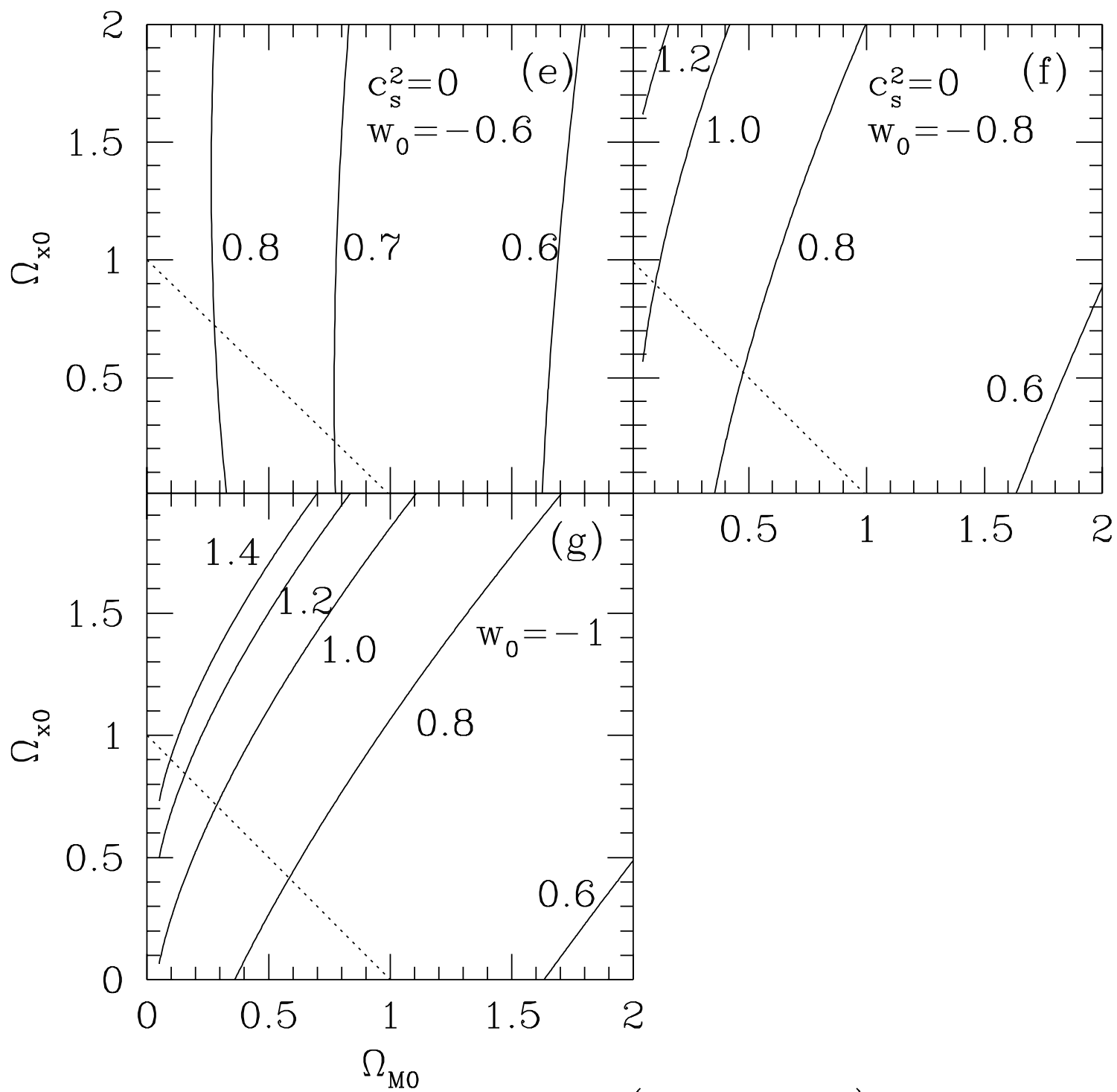


Fig.5(continued)

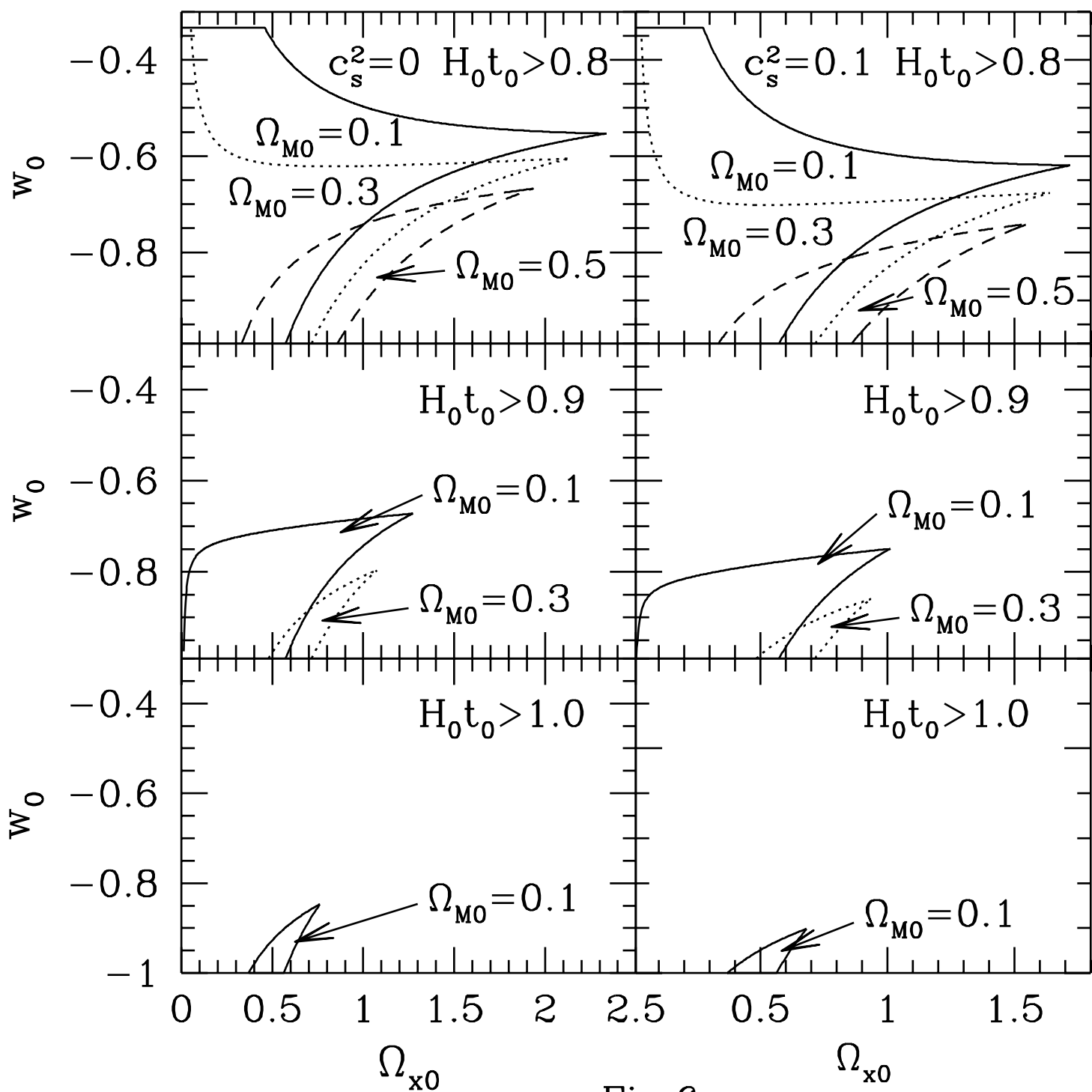


Fig.6

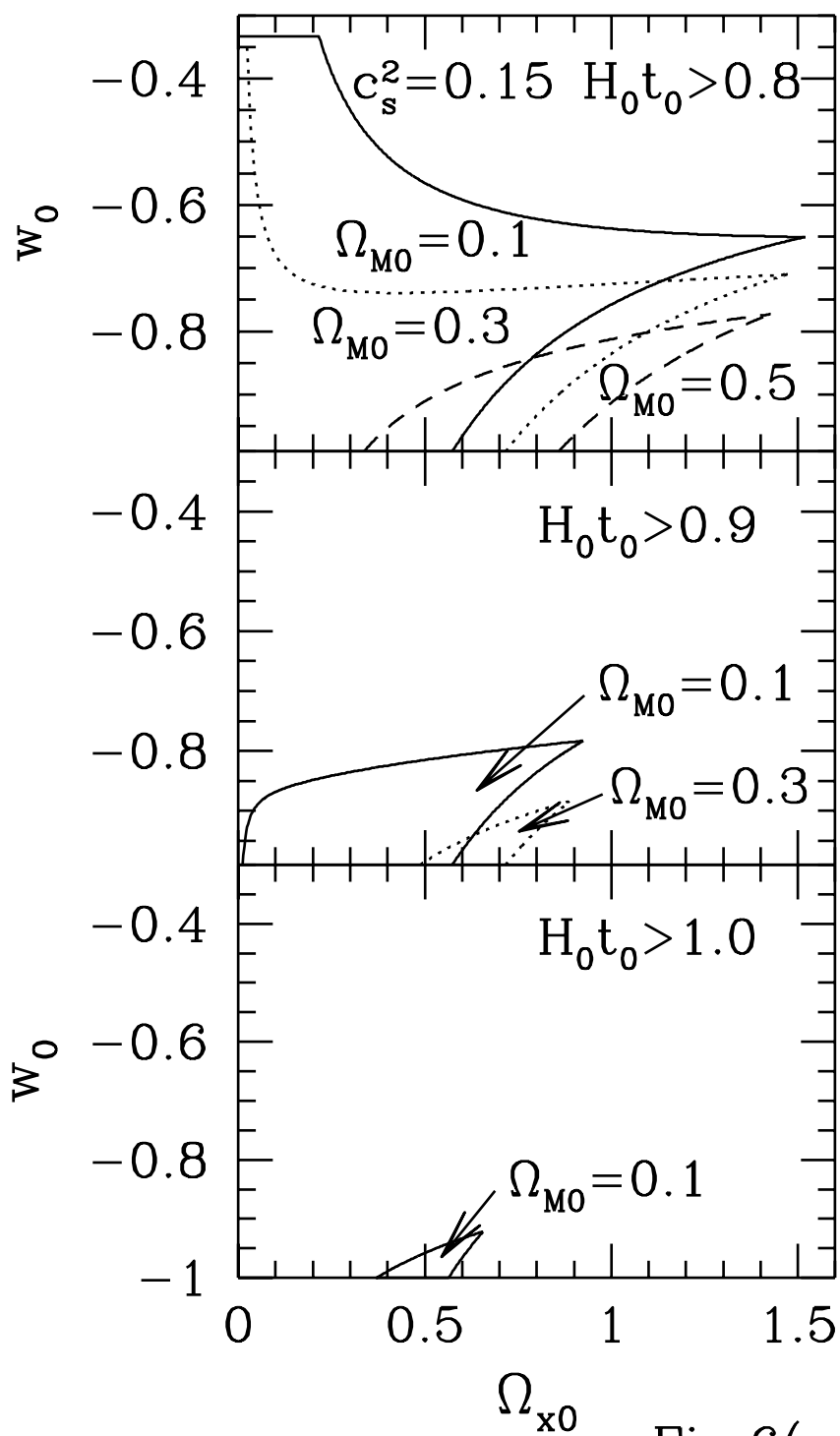


Fig.6(continued)